THE HUMMING TELEPHONE,

A Contribution to the Theoretical and Practical Analysis of its Behavior.

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The following paper describes the salient features of an experimental research on the humming telephone, conducted in the Graduate School of Applied Science of Harvard[1] University during the year 1907–08, and discusses an elementary mathematical theory which the observations appear to indicate and support.

Definition.—A "humming telephone" is a connection of:

1. A telephone receiver, or ordinary hand 'phone.

2. A telephone transmitter, or ordinary carbon microphone.

3. A source of electric power, such as a voltaic battery and telephone induction coil, with the receiver in such electric and acoustic relation to the transmitter, that it is able to emit a sustained note or hum. This auto-excited hum may be so loud as to be heard in a distant room through several partitions.

Historical Outline.—The fact that a telephone receiver held, either in contact with, or close to, the face of its transmitter may cause the production of a hum or singing tone, appears to have been first observed by Mr. A. S. Hibbard.[2] This experimental fact is now well known to telephonists. In many cases, it is only necessary to lift a subscriber's telephone from its hook, and hold it face to face with its transmitter, in order to produce a loud hum.

The only published investigation of the humming telephone that the authors have succeeded in finding is an important paper by Mr.

[1] "Investigation of the Phenomena of 'The Humming Telephone,'" by Walter L. Upson, a thesis towards the degree of master of science in electrical engineering, Harvard University, 1908.

[2] September, 1890. See Gill's paper hereafter referred to.

F. Gill,[3] read before a meeting of the Dublin Local Section of the Institution of Electrical Engineers in April, 1901. Very briefly, the salient experimental facts reported in this valuable paper are:

1. The reversal of the telephone receiver connections in the circuit alters the pitch of the auto-excited tone, the pitch being higher for one direction, and lower for the other direction, of connection.

2. The pitch of the tone may also be altered by changing: (*a*) the inductance, capacity or resistance of the circuit, or circuits; (*b*) the strength of current in the microphone transmitter; (*c*) the distance between the receiver and transmitter diaphragms; (*d*) pressure on either of the diaphragms.

The Gill paper does not discuss the theory of the subject beyond suggesting that the phase retardation of the acoustic impulses reaching the transmitter from the receiver has a controlling influence on the pitch of the tone.

The research reported in this paper may be regarded as extending the investigation from the stage reached in Gill's paper to a stage which admits of a first approximation theory. A large amount of research remains, however, to be carried on in the future, before the experimental and theoretical analysis of this fascinating but complex phenomenon can be regarded as satisfactorily nearly complete.

Method of Observation Employed.—As pointed out in Gill's paper, the pitch of the note emitted by the humming telephone, although substantially constant under fixed conditions, is affected by almost any change in the apparatus, in a seemingly most intricate manner. In order, therefore, to study the effect of varying one particular variable at a time, the device was hit upon of acoustically connecting the receiver and transmitter diaphragms in a definitely controllable way by means of telescoping tubes fitting on to the receiver and transmitter faces. These tubes, and also the standard electric connections employed, are indicated in Fig. 1.

The transmitter was kept stationary, with one end of the tube covering and secured to its cone. The receiver was fastened, on a sliding wooden carriage, to the other end of the telescoping tube.

[3] "Note on a Humming Telephone," by F. Gill, *Journal of the Institution of Electrical Engineers*, 1901–02, Vol. XXXI., No. 153, pp. 388–399.

The distance between the faces of the two instruments could be varied at will by pulling out, or pushing in, the telescoping tube-sections. The average current in the primary circuit was measured with a Weston d.c. milliammeter. The pitch of the humming note was measured approximately by the ear, with the aid of a number of short organ pipes, and, in some instances, with the aid of a violin. The voltaic battery used consisted of a selected number (from two to nine, but usually four) of 25-ampere-hour lead storage cells. The reversing switch in the secondary circuit enabled the receiver terminals to be reversed at will.

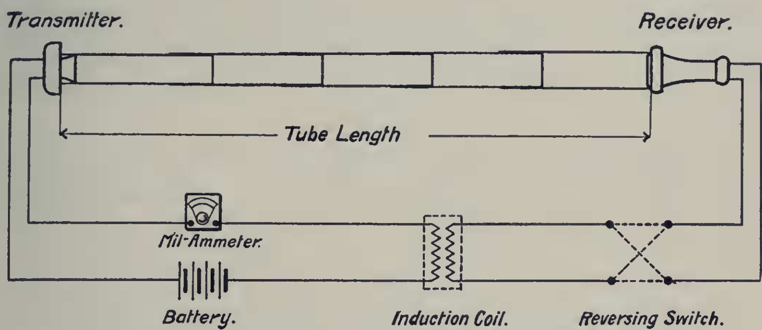

FIG. 1. Diagram of Humming Telephone Connections.

The Telescoping Tubes.—The tubes were made of heavy wrapping paper. Their internal diameters varied from 5 cm. (2 in.) to 6 cm. ($2\frac{1}{3}$ in.). They were used in lengths of 65 cm. ($25\frac{1}{2}$ in.), with a few shorter and longer sections for special measurements. The substance of which the tubes was composed did not appreciably affect the observations. It was found, however, that if the telescoping sections did not fit fairly tightly, erratic results were obtained. Closely fitting sections were used.

The Transmitters.—The transmitters used were of the standard Western Electric Co. type and manufacture. The diaphragm in these instruments was of aluminum, 6.32 cm. (2.49 in.) in total diameter, and 0.55 mm. (0.022 in.) thick, over a coating of Japan varnish on one face. The diaphragm was loaded at its center with one of the disk electrodes of the carbon microphone. The diaphragm was damped by being clamped between rubber rings to an internal diameter of 4.8 cm. (1.9 in.), and also by the application of a pair of rubber-tipped flat metal springs to areas between the center and edge. The resistance of the microphone varied between the approximate limits of 20 ohms when quiescent, and 110 ohms when in powerful vibration.

The Receivers.—The receivers used in most of the measurements were of the standard bipolar Western Electric Co.'s type, known as No. 122, having poles 1.4 × 0.2 cm. (0.55 × 0.08 in.), separated by 0.82 cm. (0.325 in.). They had a resistance of 210 ohms, and an inductance of 0.025 henry, at a frequency of 1,000 ~. With steady currents, their resistance, at 15° C., was about 70 ohms. The diaphragm of varnished ferrotype iron had an external diameter of 5.5 cm. (2.17 in.), a clamping diameter of 4.95 cm. (1.95 in.) and a thickness, over varnish, of 0.292 mm. (0.0115 in.). Its weight was 4.0 grammes.

The Induction Coil.—The induction coil used was of the standard Western Electric Co.'s type, known as No. 13. Its resistances and inductances were taken as follows:[4]

TABLE I.

Frequency Cycles per Second.	Resistance at 18° C.		Self-Inductance.		Mutual Inductance, Henrys.
	Primary Ohms.	Secondary Ohms.	Primary Henrys.	Secondary Henrys.	
0	1.62	20.3			
1,000	3.2	48.1	0.0044	0.0765	0.0172

The principal dimensions of the coil were: Length over all 8.2 cm. (3.16 in.). Interflange 6.3 cm. (2.5 in.). Diameter over outside cover 2.5 cm. (1 in.). Internal diameter of core tube 0.75 cm. (0.296 in.). Diameter of iron wires in core 0.0356 cm. (0.014 in.). Total number of iron wires in core about 75.

Observation Series No 1. Effect of Shortening the Tube.—Commencing with the connections of Fig. 1, a battery of 8.6 volts, and a tube length of 267 cm., as indicated in Fig. 2 on the scale of abscissas, a loud steady note between G″♯ and A″ (850 ~) was sustained in the telephone. The pitch of this note is shown at *P* on the

[4] The data for the coil at 1,000 ~ were kindly supplied by the engineering department of the Western Electric Co.

upper ziz-zag line *I*. The current strength, on the d.c. milliammeter, as shown at *p* on the lower ziz-zag line *I*, was 130 milliamperes. When the telescopic tube was gradually shortened, the pitch of the note steadily rose, until it reached *Q*, *at* A″♯ (920∼), with 240 cm. of tube-length, and a primary current strength *q* of 200 mas. The intensity of the note near 920 ∼ was ordinarily somewhat weaker than when near 825 ∼. On continuing to shorten the tube, the pitch suddenly broke from *Q*, at 920 ∼, to *R* at 825 ∼. Pushing

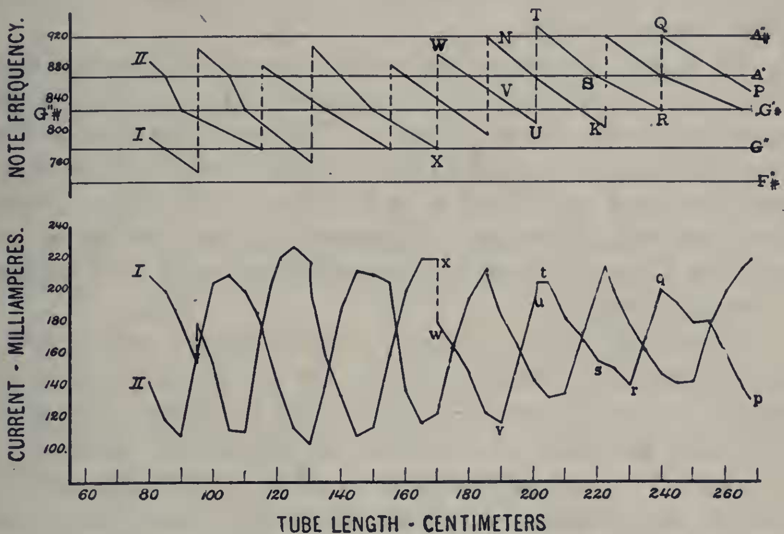

Fig. 2. Effect of Shortening Tube, and of Reversing Receiver Connections.

in the tube further, the pitch would again climb steadily to *T*, at 201 cm., with a new maximum of current. Beyond this point, the pitch would break suddenly to *U* at 810 ∼. Again it would climb to *W*, at 170 cm. and suddenly collapse to *X*. Continuing in this manner, the pitch would alternately rise to maxima and break suddenly to minima, along the pitch zig-zag *I*. At the breaks of pitch, the current would sometimes break to a lower value, as at *t*, *u*; or break to an upper value, as at *w*, *x*; or vary suddenly in rate of change, without discontinuity in magnitude, as at *q*. Repeating the experiment, the zig-zag lines of pitch and of current would be repeated, not exactly but substantially, the variations being due not merely to observational error, but also to variations in the behavior of the transmitter.

The zig-zag pitch line *PQRST* is found to be somewhat irregular. The slants are by no means regularly parallel. The breaks *QTW* are neither regularly elevated, nor regularly spaced. The only substantial regularity is in the spacing along the pitch line G″♯ of 825 ∼. The intersections of the ascending branches with this line lie approximately 40 cm. apart, at 110, 150, 190, 230 and 270 cm., or in accordance with the series $30 + 40m$ cm., where m is any positive integer.

As regards the current curve *pqrst*, its points of minima *p, r, v*, etc., correspond fairly well to the ascending intersections of the pitch line with the line of G″♯ 825 ∼. The points of maxima *q, t, x*, etc., occur near to the breaks in the pitch *Q, T, W*, etc. Minimum primary current was noticed to be associated with maximum microphonic activity of vibration. Feeble action in the microphone, on the other hand, was found to be associated ordinarily with increase of primary current.

Observation Series 2. Effect of Shortening the Tube with Reversed Receiver Terminals.—Curves *II*, in Fig. 2, represent the behavior of note pitch and primary current, as the tube was shortened from 265 cm. to 80 cm., with the terminals of the receiver reversed. Their general characters are similar to those of curves *I*. The two sets of curves indicate the effect which would be produced by reversing the receiver terminals at any particular tube-length within the above range. Thus, at *S*, or 220 cm., a reversal would lower the pitch from 870 ∼ on curve *I* to *K*, at 810 ∼, on curve *II*. On the other hand, a reversal made on curve *I*, at *V*, of 825 ∼, would raise the pitch to *N* of 900 ∼ on curve *II*, so that whether the reversal produces a rise or fall of pitch depends, in general, upon whether the reversal is effected above or below the mean pitch of G″♯, 825 ∼.

The only apparent regularity in the pitch line *II* lies in the spacing of the ascending intersections with the line of mean pitch G″♯ (825 ∼). These occur near to 90, 130, 170, 210 and 250 cm. of tube-length, or according to the series $10 + 40m$ cm. On the mean-pitch line, the ascending intersections of one curve lie approximately 20 cm. from, or midway between, those of the other curve.

The note frequencies and primary current strengths for tubes of less than 60 cm. in length are given in Fig. 3, commencing at 60 cm. and shortening down to about 1 cm., when the receiver face came into contact with the transmitter face (cone removed), and so prevented closer approach. Curves *I* and *II* of Fig. 3 correspond

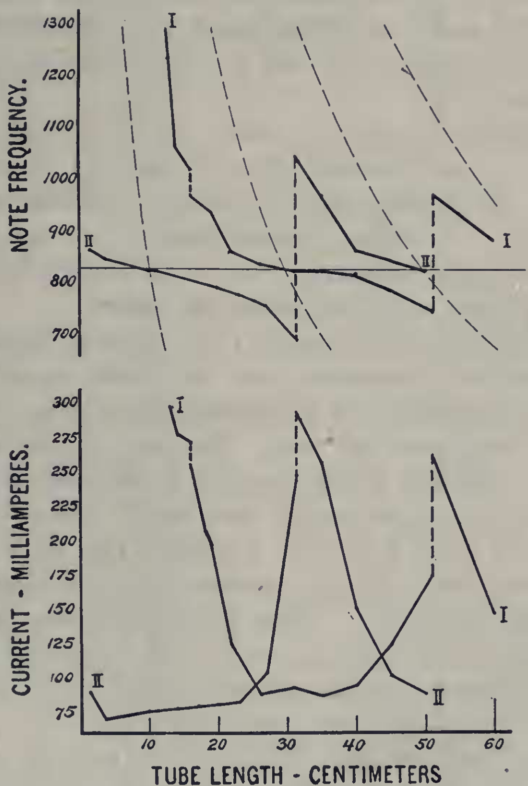

FIG. 3. Humming Note Frequencies and Primary Current Strengths with Short Tubes.

to curves *I* and *II* of Fig. 2, respectively, and indicate the effect of reversing the receiver terminals. It may be observed that following the pitch line *I*, the ascending branch intersects the mean frequency line of 825 ~, at a tube-length of 30 cm., for the last time.

Shortening the tube beyond this point, the pitch rises until it reaches e''' of 1,300 ~, at 12.5 cm., and at a primary current strength of 300 mas. Here the note breaks without descending to a new low note. There is silence with this connection of the receiver between 12.5 cm. and 0 cm. With the transmitter and receiver touching each other, it was possible to produce almost any note between 620 ~ and 1,300 ~, by giving suitable opening to the air at one side. If, however, the outside air was shut off, and the air between the transmitter and receiver diaphragms was cylindrically enclosed, by bringing their faces into full opposition and contact, no note could be obtained.

If we follow pitch curve *II*, we find that the ascending branches intersect the mean-frequency line at 50 cm. and at 10 cm. The pitch 866 ~ was obtained steadily when the transmitter and receiver faces were in full contact, corresponding to a "tube-length" of 1 cm. With this connection of receiver terminals, no other note, or variety of notes, could be obtained at contact.

A telescoping tube of 9 meters (29.5 ft.) total length was used in one series of measurements, and the results appear in Fig. 4. They were all obtained with diminishing tube-lengths, or with compression of the telescoping tube. The small crosses indicate discontinuities produced at the removal of sections of tube when finished with. In regard to the pitch line, it will be seen that it corresponds to curve *I* of Figs. 2 and 3. That is, it crosses the mean-frequency line of 825 ~ ascendingly at $30 + 40m$ cm. with a fair degree of precision. With the shortest tube, the range in pitch-frequency was from 740 ~ to 1,060 ~, or through 320 ~. At the full length of 9 meters, this range fell to 75 ~. The ultimate limit tended apparently to the mean-pitch frequency of G''♯ 825 ~. The average note was above this pitch; but this was probably because the tube was being compressed. Reference to Fig. 5 will show that, when shortening the tube, the average pitch lies above the mean of 825 ~; while in lengthening the tube, the average pitch lies below.

The primary current strength in Fig. 4 tends, in general, to minima at the mean-frequency pitch of 825 ~, and to maxima at the breaks. The differences in current strength become, however,

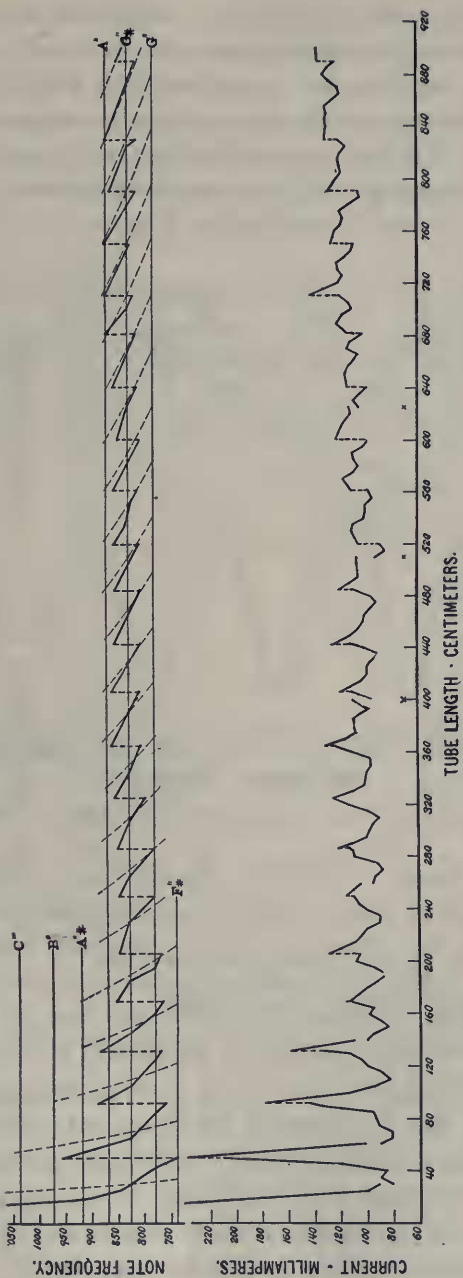

FIG. 4. Frequency and Primary Current Curves for Tube-lengths up to 920 cms. Steadily Reduced in Length, 8.6 Volts in Primary Circuit.

less marked as the tube is longer, the minimum currents rising, as the length increases, by about 40 mas. in 9 meters, indicating steadily reduced action in the transmitter with increasing distance. Since the current rose to 260 mas. when the transmitter diaphragm was entirely out of action, we should expect, at this rate, to be able to sustain the humming note to a total tube-length of 40 meters; but no tests were actually made beyond 9 meters.

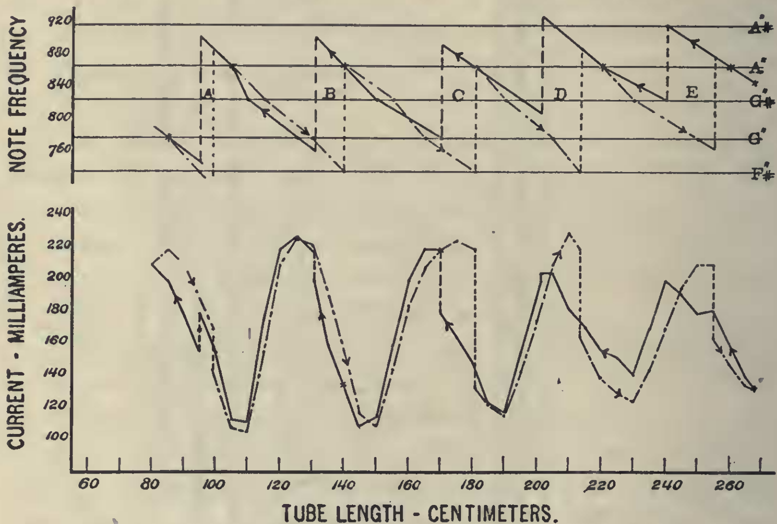

FIG. 5. Effect of Lengthening and Shortening the Tube.

Observation Series 3. Effect of Lengthening the Tube.—Fig. 5 indicates the relative effects produced by lengthening, as compared with shortening, the telescoping tube joining the transmitter and receiver in Fig. 1, using the same apparatus and connections as in Figs. 1, 2, 3 and 4. The heavy or continuous lines in Fig. 5 show the effects of shortening the tube, or correspond to curves I in Fig. 2. The broken lines show the effects of lengthening the tube. It will be observed that the points of maximum and minimum current agree fairly well. The ascending intersections of the pitch lines with the mean-frequency line of $G''\#$ 825 $\sim$, lie near together, and approximately conform to the series $30 + 40m$ cm. of tube-length. The points of break in pitch do not, however, agree, and the distances between corresponding pairs of breaks in pitch increase as the tube-length is greater, being 4 cm. at A, 9 at B, 11 at C, 13 at D, and 15 at E. Although not shown in Fig. 5, owing to limitations of space, it was found that these distances between corresponding breaks continued to increase until they reached about 20 cm., after which they shortened again to commence a new expanding series.

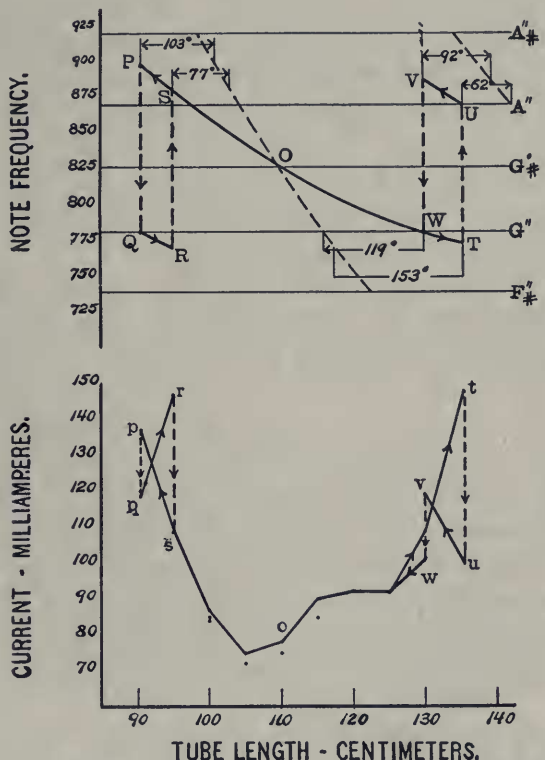

FIG. 6. Humming Cycles with Cyclic Changes in Tube-length.

Observation Series 4. Effect of Alternately Reversing, or Reciprocating, the Motion of the Tube. Humming Cycles.—If, when compressing the telescopic tube, and when the note broke from a higher to a lower pitch, the tube was immediately extended again, the note would continue to lower in pitch for a little while, and then break back to a higher pitch. By moving the tube in and out, like a concertina, over this range, the pitch would break to and fro in a very regular way. The corresponding reverse action would also occur if the motion commenced with extension. These conditions are shown in Fig. 6. Commencing at the point O, with 110 cm. of tube-length, on the mean frequency of 825 ∼, if we shorten or compress the tube to 90.5 cm., we reach P at 900 ∼, near A″♯. The note then breaks to Q at 780 ∼. Increasing the tube-length back to 95 cm., we reach R at 770 ∼. The note then breaks upwards to S at 880 ∼. This humming cycle $PQRS$, could be repeated indefinitely with a considerable degree of precision as to pitch and tube-length; but with a more moderate degree of precision as to primary current strength. Similarly, the cycle $TUVW$, of 10.5 cm. amplitude in length, and 100 ∼ amplitude in pitch, might be repeated indefinitely. The amplitudes and areas of these humming cycles vary at different breaking points.

Purity of Humming Tone.—With the greater tube-lengths, shortly before the break of pitch occurred, there was frequently noted an appearance of the new tone in advance. As the breaking point was approached, the old tone dwindled, while the new tone strengthened. At the break, the old tone, already faint, would suddenly cease. Consequently, before breaking, both the old and new tones might be recognized, forming a sort of trill, or combination tone. This association of simultaneous tones had the effect of maintaining the primary current strength more nearly uniform. With the shorter tube-lengths, which involved a greater jump of frequency at the breaks, these combination tones were rarely heard, and the old note would break suddenly into the new note without any suggestion of a trill.

In some of the observations, the notes, aside from the above-mentioned trilling near to the breaking points, gave acoustical evidence of multiple tones. Occasionally, the principal tone was accompanied by an octave overtone. The octave might be either the first octave below, or the first octave above, the principal tone. Such overtones were comparatively faint. At other times, the superposed tone, instead of being harmonic to the principal tone, appeared to differ therefrom by only about one tone on the musical scale. This inharmonic superposed tone was also relatively faint with respect to the principal tone. Generally, however, no superposed tones could be discerned, and the note was clear and flute-like in quality. Irregularities in the fitting of the telescoping tube-sections, or in other acoustic connections, were found to be productive of superposed notes.

Effects of Electrical Changes.

Observation Series 5. Effect of Resistance in Primary or Secondary Circuit.—In this test a single tube of constant length (86.5 cm. or 34 in.) was used. It was of pasteboard, had an internal diameter of 5.1 cm. (2 in.) and weighed 113.5 gm. This length happens to be about midway between the ascending intersections of pitch lines *I* and *II* in Fig. 2 measured on the mean-frequency line of 825 ∼. That is, the tube-length selected favored each of the lines *I* and *II* nearly equally. The battery e.m.f. of 8.6 volts was the same as in all the above described measurements. The same telephone receiver and induction coil were also used. Substantially non-inductive resistance was introduced, by rheostat, into either the primary, or the secondary, circuit at will, leaving the connections of Fig. 1 otherwise unchanged.

After starting the loud humming note with no extra resistance in either circuit, resistance was gradually inserted into the primary circuit until the note, diminishing in amplitude, finally disappeared. The extra resistance in the circuit at the extinction of the tone was recorded, under the name of "*extinguishing resistance.*" Resistance was then withdrawn from the primary circuit, and, after the loud note had been reëstablished, was introduced gradually into the secondary circuit, until again the note was extinguished. The secondary extinguishing resistance was likewise recorded. The same tests were repeated with the telephone receiver terminals reversed.

It was found that both the primary and secondary extinguishing resistances repeated themselves very fairly (within about 5 per cent.) in successive trials. In order to obtain the best comparative results in successive tests, it was found desirable to tap the transmitter gently when approaching the condition of extinction.

The pitch of the tone when enfeebled almost to extinction by extra resistance, in either the primary or secondary circuit, was always close to the mean frequency of 825 ∼.

The amount of either the primary or secondary extinguishing resistance was found to depend upon the adjustment and operative condition of the transmitter, keeping the receiver, tube-length and all other conditions unaltered. This led to a trial of this method as a practical test of microphone transmitters.

Observation Series 5a. Test of Transmitter by Hum-extinguishing Resistances.—A number of transmitters, some good and others imperfect, were tested under the conditions above outlined. These transmitters were kindly loaned for this purpose by the Western Electric Co. Twelve were regular standard instruments that had already satisfactorily passed the factory tests. These were labelled T_1 to T_{12} respectively. Four more were marked defective and "down in volume." They were labelled T_{13}, T_{15}, T_{16} and T_{24}. Four more were marked defective and "thick in quality." These were labelled T_{14}, T_{17}, T_{22} and T_{23}. Yet another four were marked defective and "burning." These were labelled T_{18}, T_{19}, T_{20} and T_{21}. Defective transmitters "down in volume" are recognized as weak. Those which are of "thick quality" are strong but defective in articulation. Those which are "burning" produce slight arcing, at or near the electrodes, when subjected to normal conditions of operation.

The results of the tests on these 24 transmitters are given in the accompanying table; where R represents the primary, and r the secondary, extinguishing resistance, when the transmitter was gently tapped. Care was taken that the observer in this test did not know the label number, or reported condition, of the transmitter under trial. It will be seen that with the good transmitters, the mean primary extinguishing resistances were all included between 26.5 and 58.5 ohms, their mean secondary extinguishing resistances being between 1,925 and 4,150 ohms. All of the defective transmitters lay outside these limits, the "down in volume" being low, and the "thick quality" high, in their extinguishing resistances; except two of the "burning" type, which fell within the good secondary extinguishing resistance limits. It would seem, therefore, that this resistance method constitutes a possible practical application of the humming telephone to transmitter testing; except that "burning" transmitters may require a separate test for their detection.

The results recorded in the last two columns of Table II. are presented graphically in the target diagram of Fig. 7. The square includes all the good instruments and none of the bad. The mean of the good transmitters is indicated by the solid black circle.

TABLE II.

Table of Comparative Hum-Extinguishing Resistances for 12 Good and 12 Defective Transmitters.

Quality of Transmitter.	Transmitter.	Extinguishing Resistances. 1st Position of Rec'r. R (Pri.).	1st Position of Rec'r. r (Sec.).	2d Position of Rec'r. R (Pri.).	2d Position of Rec'r. r (Sec.).	Average. R.	Average. r.
OK.	T_1	43	3,100	50	2,300	46.5	2,800
	T_2	23	3,050	44	3,000	33.5	3,025
	T_3	55	4,300	43	4,000	49	4,150
	T_4	25	2,300	31	1,900	28	2,100
	T_5	45	3,900	40	2,600	42.5	3,250
	T_6	31	3,000	69	3,800	50	3,400
	T_7	27	2,600	26	1,600	26.5	2,100
	T_8	30	2,600	30	1,700	30	2,150
	T_9	31	2,700	44	2,300	37.5	2,500
	T_{10}	47	3,900	46	3,900	46.5	3,900
	T_{11}	43	4,100	74	3,800	58.5	3,950
	T_{12}	33	2,600	21	1,250	27	1,925
	Mean	36.1	3,180	43.2	2,696	39.6	2,937
Down in Volume.	T_{13}	10	750	10	700	10	725
	T_{15}	6	390	—	—	6	390
	T_{16}	6	900	12	1,000	9	950
	T_{24}	17	1,400	20	1,500	18.5	1,450
	Mean	9.75	860	14	1,070	10.9	879
Thick Quality.	T_{14}	62	4,900	66	2,300	64	3,600
	T_{17}	50	9,000	70	10,000	60	9,500
	T_{22}	52	7,000	75	8,000	63.5	7,500
	T_{23}	66	5,300	78	6,700	72	6,000
	Mean	57.5	6,550	72.5	6,750	64.75	6,650
Burning.	T_{18}	54	3,800	68	3,400	61	3,600
	T_{19}	61	4,700	93	6,900	77	5,800
	T_{20}	82	5,300	102	5,000	92	5,150
	T_{21}	60	3,900	61	2,900	60.5	3,400
	Mean	64.25	4,425	81	4,550	72.6	4,487

Observation Series 6. Effect of Varying the E.M.F. in the Primary Circuit.—Among so many variables and variations as are displayed in preceding diagrams, it is comforting to find one variable which produced relatively little effect within certain practical limits. Fig. 8 shows the frequencies and primary currents for tube-lengths

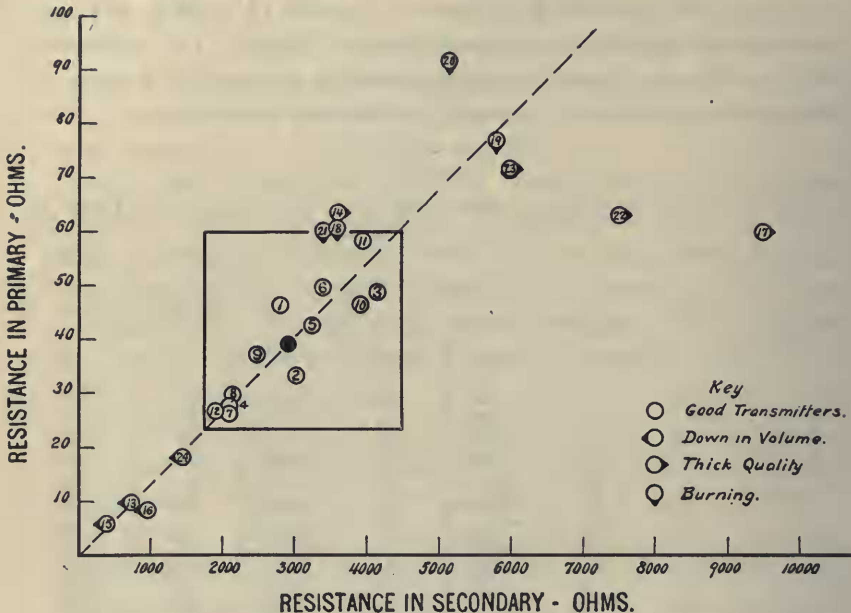

FIG. 7. Target Diagram of Transmitter Tests by the Method of Hum Extinguishing Extra Resistance.

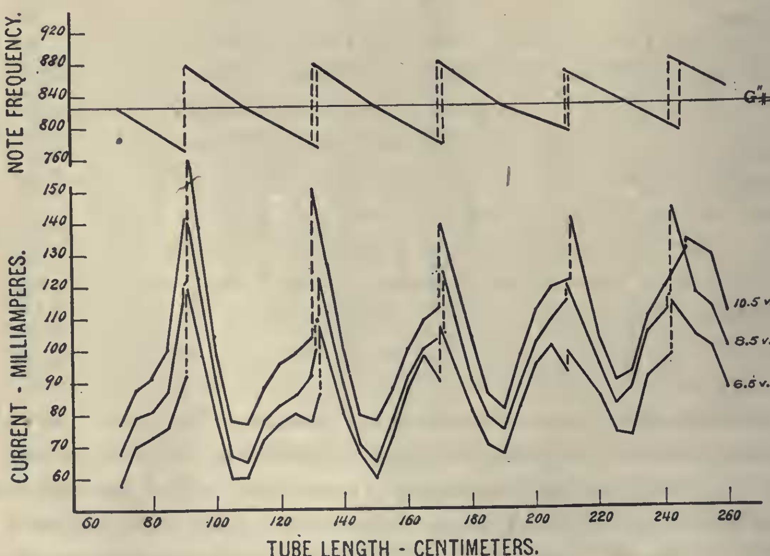

FIG. 8. Frequencies and Primary Currents for Different Primary E.M.F's.

steadily reduced from 260 to 70 cm., with batteries of 3, 4 and 5 storage cells, respectively, in the primary circuit (6.5, 8.5 and 10.5 volts). The transmitter, induction coil, receiver and transmitter were all as in Figs. 1 to 6. It will be seen that the primary currents have their respective maxima and minima in substantial agreement, the range of variation being naturally greatest for the largest battery, and least for the smallest. The ascending intersections of the frequency line with the mean-frequency line of 825 ∼ are the same throughout, and conform to the series $30 + 40m$ cm., in agreement with line *I* of Fig. 2. The breaks in pitch do not all coincide; but the differences in this respect are not great, nor can it be said

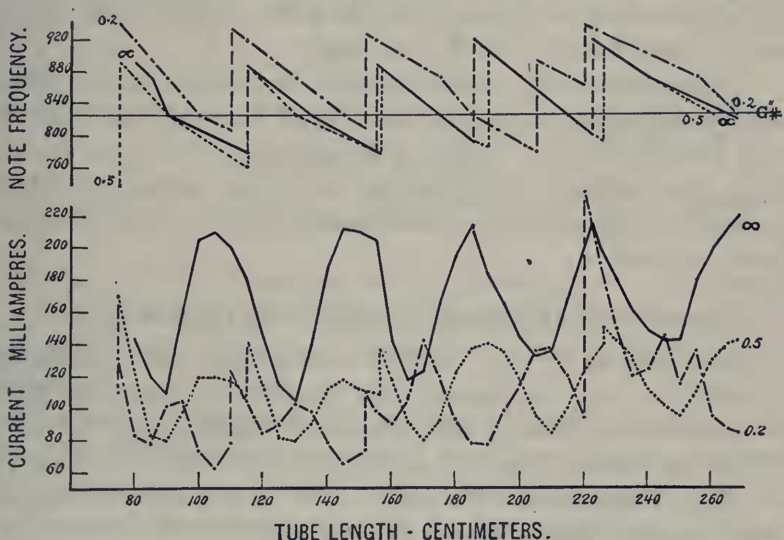

FIG. 9. Frequencies and Primary Current Strengths for Different Condensers in Secondary Circuit.

that the biggest battery always produced the most retarded break. Moreover, excepting perhaps the break at 240 cm., the variations in breaking points are within the limits of variation obtained in successive series with one and the same battery.

Observation Series 7. Effect of a Condenser in the Secondary Circuit.—It was found that a certain magnitude of condenser capacity inserted in series in the secondary circuit had a marked effect on the behavior of the humming telephone. The results are indicated in Fig. 9, for a tube-length commencing at 270 cm. and steadily reduced to 75 cm., with 8.6 volts in the primary circuit and the same instruments as before. Three sets of curves are given, for 0.2 μf. (microfarad), 0.5 μf., and ∞ μf. (condenser short-circuited), respectively. Referring to the pitch lines, it will be seen that there is not much difference between the cases of ∞ and 0.5 μf. The ascending branches of the zig-zags cut the mean frequency line of G″♯ at 90, 132.5, 175 and 210 cm. or fairly in conformity with the series $10 + 40m$, as in curve *II* of Fig. 2. With 0.2 μf., however, the intersections with this line are at 100, 145 and 185 cm., or more nearly in conformity with the series $22 + 40m$ cm.; that is, at points displaced about 12 cm. further along the tube. Moreover, the breaks occur at higher frequencies by about 40 ∼.

As regards primary current strengths, the minima in each series occur at substantially the points where the pitch line intersects ascendingly with the G″♯ line. That is, the minima of ∞ and 0.5 μf. are fairly close together; while those for 0.2 μf. are displaced about 12 cm. further along the tube. Maximum currents occur near breaking points, as usual.

Effects of Mechanical Changes in Instruments.

Observational Series 8. Effects of Modifying the Transmitter.—In order to study the influence of changes in the transmitter upon the humming note, three similar Western Electric transmitters were selected, of standard type and quality, already referred to as T_5, T_8 and T_{11}, in connection with Fig. 7. The receiver, induction-coil, battery and connections were as in previous tests. The comparative results with these three transmitters are shown in Fig. 10, for tube-lengths steadily reduced from 260 to 70 cm. It will be noted that the ascending intersections of the pitch lines all intersect the mean-frequency line of 825 ∼ in substantial conformity with the series $30 + 40m$, or in accordance with curve *I* of Fig. 2. The breaking points do not agree, No. 8 always breaking last at a higher pitch, No. 5 next at a medium pitch and No. 11 first at a lower pitch. It may also be noted that in the hum-extinguishing resistance-test of these three transmitters, as given in Table II., and in Fig. 7, their order of succession was the same.

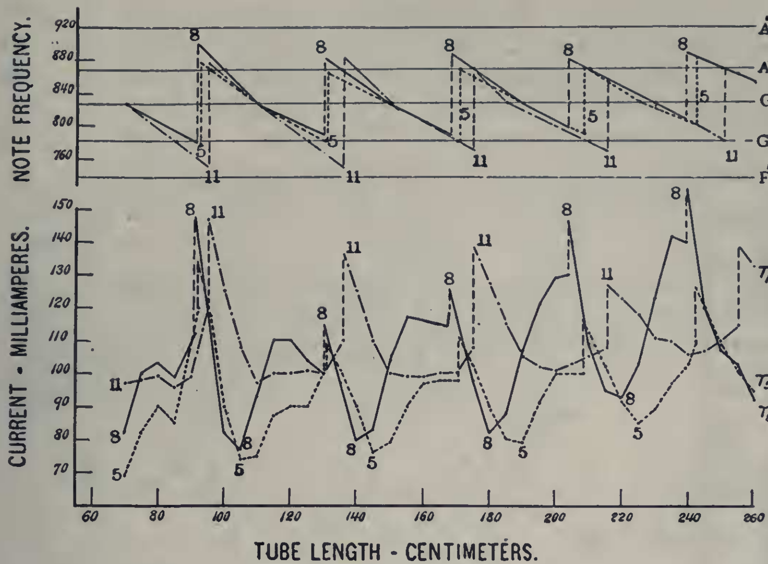

FIG. 10. Comparative Behavior of Three Regular Standard Transmitters with Reduced Tube-lengths.

The test indicates, therefore, that different standard transmitters in normal adjustment do not alter the mean-frequency tube-lengths; but that variations in breaking lengths may be expected within certain limits.

A further test was made of the effect of modifying the transmitter, by selecting for experiment a particular Western Electric Co.'s standard type of transmitter which had been used in the laboratory for some years, and was not in the best adjustment. A test was made with this instrument (using the same receiver, coil, battery and connections as in preceding tests), first without any extra load on its diaphragm, second with a load, and third with the load removed. The load consisted of a small brass disk 1.5 cm. (0.59 in) in diameter, and 0.2 cm. (0.079 in.) thick, clamped at its center between the two small nuts at the center of the external surface of the diaphragm. This added a mass of 2.7 gm. to the vibrating system of the transmitter. The results are seen in Fig. 11. Curves 1 and 3 represent the behavior of the system unloaded, before and after loading respectively, the tube-length being steadily diminished from 200 to 80 cm. Curves 2 represent the corresponding behavior when the diaphragm was loaded. The primary currents were all unusually large, probably owing to the imperfect adjustment of the transmitter.

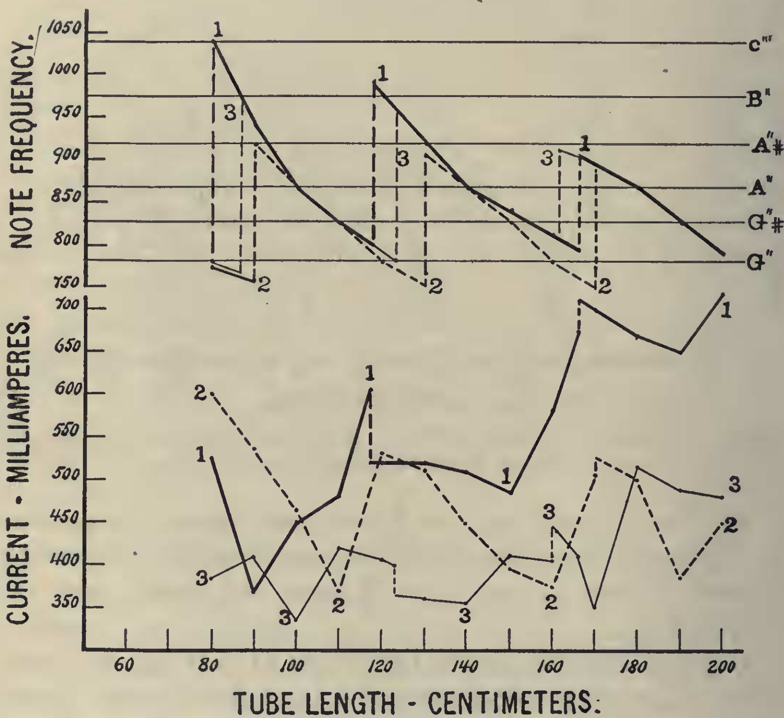

FIG. 11. Test of a Transmitter with its Diaphragm Loaded and Unloaded.

It will be observed that the loading did not appreciably alter the ascending intersections of the pitch lines with the G″# mean-frequency line, which occur in conformity with the series $20 + 40m$ cm. The loading seems to have somewhat lowered the range of pitch as a whole; or to have modified the conditions at breaking, without materially affecting the conditions at mean-frequency (825 ~).

A number of trials with further modifications of the transmitter diaphragm substantiated the above stated results. In one case, a new experimental diaphragm of tinned sheet iron, 0.38 mm. thick

(0.015 in.), with parallel and opposite symmetrical sectors sliced off, was substituted for the regular diaphragm in the test transmitter. The primary current strength during activity was thereby increased; but the G″♯ tube-lengths remained substantially unchanged at $30 + 40m$ cm. Adding loads, altering the damping-spring pressure, or varying the other mechanical adjustments of the transmitter produced either complete silence; or else the usual G″♯, at $30 + 40m$ cm.

The tests showed that modifying the transmitter alters the range and limits of pitch variation, as well as the primary current strengths; but does not sensibly alter the tube-lengths for mean-frequency.

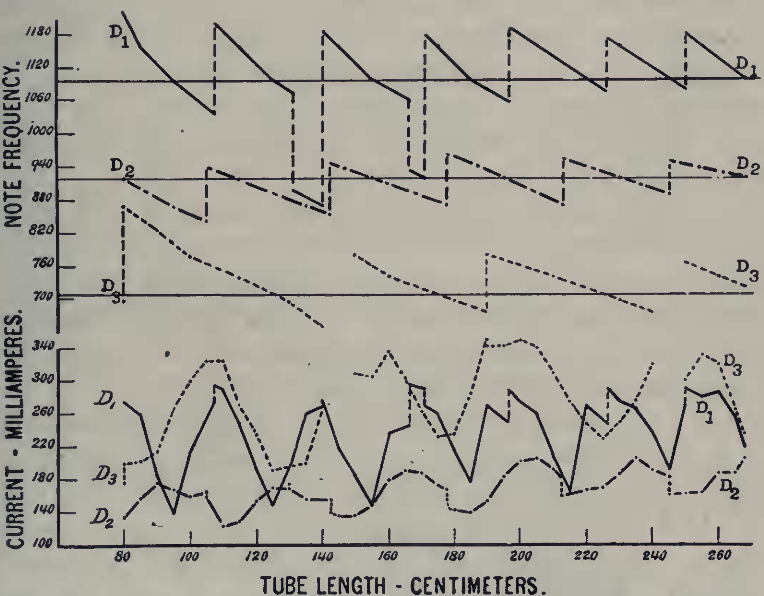

FIG. 12. Comparative Frequencies and Currents with Three Different Receiver Diaphragms.

Observation Series 9. Effect of Altering the Receiver.—In order to determine the influence of the telephone receiver diaphragm on the hum, three special receiver diaphragms were made up, each of soft transformer steel, 0.355 mm. (0.014 in.) thick, and 5.5 cm. (2.16 in.) in diameter, labeled D_1, D_2 and D_3 respectively. D_1 was left circular, D_2 and D_3 had symmetrical sectors cut from opposite sides, reducing their width to 4 cm. (1.57 in.) and 3 cm. (1.18 in.) respectively. In clamping these strip diaphragms in front of the bipolar magnet of the standard receiver, their angular position did not appear to affect the system appreciably.

The results obtained with these three diaphragms are indicated in Fig. 12, for tube-lengths diminished steadily from 270 to 80 cm. It will be seen that the receiver diaphragm influences the hum profoundly. Thus, the circular diaphragm D_1 developed a mean-frequency of 1,100 ~ or c'''♯, judging by the points of minimum primary current, and its pitch zig-zag formed ascending intersections with this line at 95, 125, 155, 185, 215 and 245 cm., approximately, in conformity with the series $5 + 30m$ cm. The sectored diaphragm D_2 developed a mean-frequency of A''♯, at 920 ~, with ascending intersections nearly in conformity with the series $36m$ cm. The narrowest diaphragm D_3 developed a mean-frequency of F'', at 705 ~, and ascending intersections in substantial conformity with the series $33 + 47m$ cm.

It will be observed that there are double breaks in pitch on zig-zag D_1. This tendency was found to follow irregularity in the diaphragm, or in its mounting. Thus, the ordinary standard diaphragm used in all the preceding tests was observed to develop similar double breaks when the clamping screw-cover was slackened, so as to leave the diaphragm somewhat loosely clamped.

The pitch zig-zag of D_3 shows gaps. These gaps seemed to be due to the enfeebled condition of the electromagnetic vibrating system in the receiver when used with the experimental diaphragm D_3. A very marked case of such gaps is presented in Fig. 13, which indicates the frequencies and currents obtained with a particular single-pole telephone receiver, the remainder of the apparatus being unchanged, and the tube-length being steadily reduced from 165 to 80 cm. The line of mean-frequency is at 1,025 ~, and the ascending intersections with this line are formed at points conforming with the series $28 + 32m$ cm. Only short pieces of the zig-zag were, however, obtainable, and these only with the aid of a condenser in the secondary circuit. The dotted segments *RS* and *TV* were obtained with the receiver terminals reversed, and correspond approximately to ascending intersections of the series $13 + 32m$ cm.

Various other modifications of receiver and receiver diaphragm were tried. Loading the diaphragm with a small central mass lowered the mean humming frequency. By selecting suitable diaphragm dimensions, the mean-frequency of the hum could be varied between wide limits.

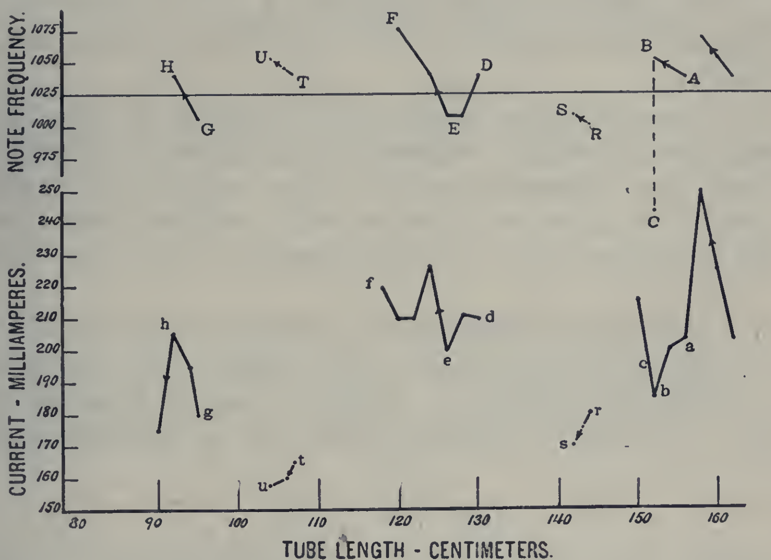

Fig. 13. Discontinuous Frequencies, or Large Gaps in Curves, for Case of Singe-pole Receiver.

Conclusions Directly Derivable from the Experiments.—The following more prominent conclusions are indicated by the experiments themselves, independently of any theory:

1. The mean-frequency of the humming-telephone note is determined solely by the receiver diaphragm, and its natural free rate of vibration.

2. The ascending intersections of the frequency zig-zag with the mean-frequency line will be formed approximately at tube-lengths of $(\frac{3}{4} + m)\ v/n_0$ cm. for one connection, and of $(\frac{1}{4} + m)\ v/n_0$ cm. for the other connection, of the receiver; where v is the velocity of sound in air (33,000 cm. per sec. nearly), n_0 is the mean frequency in cycles per second, and m is any positive integer, within

the working range of the tube. The constants $\frac{3}{4}$ and $\frac{1}{4}$ may be modified by the presence of condensers, and other circumstances.

3. The range of pitch variation, and the breaking positions, are determined by the transmitter, and by the reinforcing capability of the system. For systems that are weak, either electrically or acoustically, the range of pitch, above or below the mean, will be small.

4. The primary current, as measured by a d.c. instrument, is ordinarily a minimum at the mean frequency, and a maximum at a break.

5. Transmitters may be tested for effectiveness, by measuring their hum-extinguishing resistances in the primary or secondary circuit. The tube-length should be such as to produce mean frequency if one connection of receiver only is used, but should favor both connections equally, if both connections of receiver are used.

Outline of Theory of the Humming Telephone.

Preliminary Considerations. Simple Orbital Motion and Simple Unretarded Vibration.—Let a particle of mass m grammes describe a simple plane circular orbit zab, Fig. 14, about the center

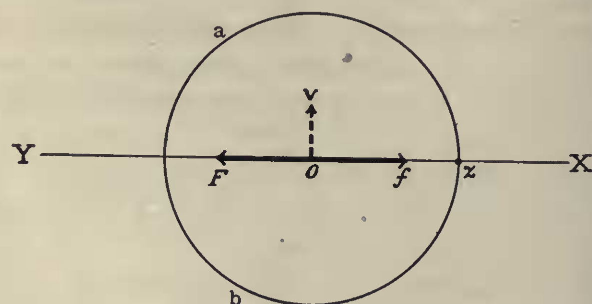

Fig. 14. Vector Diagram of Free Undamped Vibration.

O. Let the radius $Oz = r$ cm., and let OX be the initial line of reference. At time $t = 0$ seconds, let the particle occupy the position z; so that its initial radius vector is Oz. Let ω be the uniform angular velocity of the particle about the center O, in radians per second. Then, after the lapse of t seconds, the particle will occupy a point in the plane defined by the vector displacement

$$\xi = r\epsilon^{j\omega t} \qquad \text{cms.} \ \angle \qquad (1)$$

where $j = \sqrt{-1}$, and ξ is the displacement of the particle in cms. from O at the angle ωt, measured positively, or counter-clockwise, from the initial line OX.

Let the particle be acted upon by a centrally directed elastic force

$$F = -A\xi = -ma\xi = -mar\epsilon^{j\omega t} \qquad \text{dynes} \ \angle \qquad (2)$$

proportional to and opposing the displacement, as represented by the vector OF in Fig. 14. Let there be no other forces except those of inertia, acting on the particle; so that the movement is frictionless. Then the velocity of the particle at any instant t will be

$$v = \dot{\xi} = j\omega r\epsilon^{j\omega t} \qquad \text{cms./sec.} \ \angle \qquad (3)$$

The direction of the velocity will, therefore, be perpendicular to the radius vector, or parallel to the instantaneous tangent, as indicated by thè dotted line Ov, 90° ahead of Oz in phase displacement.

The acceleration of the particle will be, at any instant t,

$$c = \dot{v} = \ddot{\xi} = -\omega^2 r\epsilon^{j\omega t} \qquad \text{cms./sec.}^2 \ \angle \qquad (4)$$

That is, the acceleration will be directed oppositely to the displacement. Thus at time $t = 0$, represented in Fig. 14, the acceleration will be directed along OY. The virtual reactive force of inertia will be

$$f = -mc = -m\ddot{\xi} = m\omega^2 r\epsilon^{j\omega t} \qquad \text{dynes} \ \angle \qquad (5)$$

In Fig. 14, this reactive force of inertia is represented by Of.

In order that the circular orbital motion shall be stable, the sum of the forces OF and Of, of elasticity and inertia must be zero; or

$$OF + Of = 0 \qquad \text{dynes} \ \angle$$

$$\therefore \quad -mar\epsilon^{j\omega t} + m\omega^2 r\epsilon^{j\omega t} = 0 \qquad \text{dynes} \ \angle$$

whence

$$\omega = \sqrt{A/m} = \sqrt{a} \qquad \text{radians/sec.} \qquad (6)$$

If, therefore, the angular velocity of the motion be numerically equal to the square root of elastic force per unit of mass, the orbit will be circular and stable, and Fig. 14 may represent its vector diagram. The particle z rotates about O, at constant radius with uniform angular velocity ω, and the pair of equilibrating forces OF and Of rotate in synchronism with it. The entire system, Fig 14, may be imagined as pivoted about an axis through O perpendicular to the orbital plane, and spun about this pivot with uniform angular velocity ω.

By a well known proposition connecting simple harmonic vibration with circular orbital motion, the displacements in the former are the projections of the displacements in the latter, upon a straight line passing through the center of the system. In other words, to every case of simple circular orbital motion in two dimensions corresponds a case of simple harmonic vibration, its projection in a single dimension. Consequently, at time t, we have for the displacement in the case of simple vibration,

$$\xi = r\epsilon^{j\omega t} \qquad \text{cms.} \quad (7)$$

measured along the initial line OX by projection. The real part only of ξ is retained, and the imaginary part ignored. Similarly, the vibratory velocity will be

$$v = \dot{\xi} = j\omega r\epsilon^{j\omega t} \qquad \text{cms./sec.} \quad (8)$$

taking only the real part of the equation, or the projected value along YOX. Again, the vibratory acceleration will be

$$c = -\omega^2 r\epsilon^{j\omega t} \qquad \text{cms./sec.}^2 \quad (9)$$

retaining only the real or projected part. Similar reasoning applies to the forces of elasticity and inertia. The same equations appear as in the circular orbit case; but only their real, or horizontally projected values, are retained. Consequently, we deduce that the vibration of a particle possessing elasticity and inertia without frictional retardation will be stable and self sustained under the condition

$$\omega = 2\pi n = \sqrt{A/m} = \sqrt{a} \qquad \text{radians/sec.} \quad (10)$$

where n is the frequency of the vibration in cycles per second.

If, for example, the diaphragm of a telephone receiver had simple elasticity and inertia without frictional retardation, such that the elastic intensity $a = 26.87 \times 10^6$ dynes per cm. of displacement and per gramme mass, then any displacement released would be followed by an indefinitely sustained angular velocity

$$\omega = \sqrt{26.87 \times 10^6} = 5{,}184$$

radians per second, corresponding to $n = 825$ cycles per second. If the initial displacement were $r = 0.01$ cm., the corresponding simple circular orbit, Fig. 14, would have a radius of 0.01 cm., an angular velocity of 5,184 radians per second, an orbital velocity of 51.84 cm. per second, and an acceleration of 268,700 cm. per second. If the elastic force A were 1.3435×10^6 dynes per cm. of displacement and the effective mass were 0.05 gm., the elastic force OF would be 13,435 dynes, and the centrifugal force Of 13,435 dynes, the two being equal and in complete opposition.

Case of Free Vibration Damped and Unreinforced. Spiral Orbital Motion.—In the case of the particle moving about a center, let the motion be retarded by a force f', proportional to the velocity, defined by the relation

$$f' = -\Gamma v = -2m\gamma v \qquad \text{dynes} \angle \quad (11)$$

Then the orbital displacement at any time t becomes

$$\xi = r\epsilon^{(-\gamma + j\omega)t} \qquad \text{cms.} \angle \quad (12)$$

The orbital velocity is

$$v = \dot{\xi} = r(-\gamma + j\omega)\epsilon^{(-\gamma + j\omega)t} \qquad \text{cms./sec.} \angle \quad (13)$$

The orbital acceleration is

$$c = \dot{v} = \ddot{\xi} = r(-\gamma + j\omega)^2\epsilon^{(-\gamma + j\omega)t} \qquad \text{cms./sec.}^2 \angle \quad (14)$$

Each of the above equations defines an equiangular spiral, an inwardly directed spiral in which the curve makes a constant direction $-\gamma + j\omega$ with the radius vector.

The vector diagram for this case is indicated in Fig. 15. Let

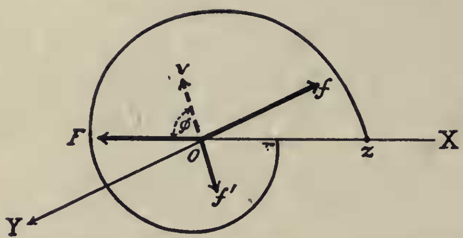

Fig. 15. Vector Diagram of Free Damped Vibration.

z be the position of the particle at any instant. The velocity at this instant will have the vector OV, parallel to the tangent at z, where

$$\tan \phi = \omega/\gamma \qquad (15)$$

The acceleration at the same instant will be directed along OY, the angles XOV and VOY being each equal to the supplement of ϕ. The virtual force of inertia will be directed along Of. The retarding force, opposing the velocity, will be directed along Of'. At any instant the vector sum of the three forces of elasticity, retardation and inertia must be zero. That is,

$$OF + Of' + Of = 0 \qquad \text{dynes } \angle$$

or

$$-mar\epsilon^{(-\gamma+j\omega)t} - 2\gamma mr(-\gamma + j\omega)\epsilon^{(-\gamma+j\omega)t}$$
$$- mr(-\gamma + j\omega)^2\epsilon^{(-\gamma+j\omega)t} = 0 \qquad \text{dynes } \angle$$

whence

$$\omega = \sqrt{a - \gamma^2} = \sqrt{\omega_0{}^2 - \gamma^2} = \omega_0 \sin \phi \qquad \text{radians/sec.} \qquad (16)$$

where ω_0 is the unretarded angular velocity. That is, the angular velocity of orbital rotation has been reduced by the retardation in the ratio of $\sin \phi$, Fig. 15, and the displacement or radius vector r continually dwindles with time by $\epsilon^{-\gamma t}$.

In the corresponding case of free damped vibration, the above equations apply; but their real parts only are taken. In Fig. 15, the projections of the vectors on a straight line through O, are selected. The dwindling vibrations of a tuning fork, or the oscillatory discharge of a condenser through a circuit containing resistance and inductance, obey this law. In the last named case, the inductance corresponds to the mass m, the reciprocal of the capacity corresponds to the elastic coefficient A, and the resistance corresponds to the velocity-resisting coefficient Γ. The condenser-charge, or electric quantity, corresponds to the vibratory displacement, the electric current to the vibratory velocity, the discharging electromotive force to the elastic force OF, the resistance e.m.f. to Of', the e.m.f. of self-induction to Of, and the impedance of the discharging circuit to the vector $m\omega_0 \angle \phi$, or $\sqrt{MA} \angle \phi = \Gamma/2 + jm\omega$.

Case of Retarded Free Vibration Reinforced. Restored Circular Orbit.—In order to sustain stable orbital motion in a particle retarded with a force proportional to the velocity, it is necessary

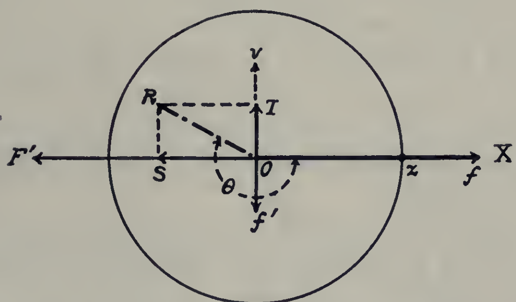

FIG. 16. Vector Diagram of Reinforced Vibration.

to supply energy continuously to the particle and to act upon it with a force equal but opposite to the velocity-resisting force. The orbit will then be restored from an inmoving spiral to a simple circle. The displacement, velocity and acceleration of the particle, Fig. 16, will then be severally expressed by equations (1), (2) and (3) applied to Fig. 14.

Let OR, Fig. 16, be an outwardly directed force from the center O, the magnitude of OR being some function $m\chi(r)$ of the radius of displacement, and θ the phase retardation behind the displacement, reckoned positively in the direction indicated, or clockwise. The restoring force $OR \angle \theta$ may be analysed into two components $OT = OR \sin \theta$, and $OS = OR \cos \theta$ along the directions OV and OX respectively. The component OT may be called the velocity component, or T component, since it acts in the direction of the velocity Ov, and against the retarding force Of'. The component Os may be called the S component, or the new elastic component. It coacts with the elastic force OF that resists displacement.

In order that the circular orbit may be retained, it is necessary and sufficient that the T component of the restoring force shall equilibrate the velocity-resisting force Of'; or, if R be the restoring force, that

$$R \sin \theta + f' = 0 \qquad \text{dynes}$$

If the T component should be less than the velocity-resisting force, the system will lose energy. The orbit will spiral inwards until the velocity has been sufficiently diminished to equilibrate the T component, and permit a stable circular orbit of reduced radius to be restored. If, on the contrary, the T component exceeds the velocity-resisting force, the system will accumulate energy, and the orbit will spiral outwards until the radius and velocity of the motion are sufficient to restore equilibrium and permit a circular orbit of enlarged radius to be maintained.

In the condition of equilibrium represented in Fig. 16, we have four forces acting on the particle, forming two separate equilibrating pairs; namely, a pair along the displacement vector Oz, which we may call the displacement pair, and a pair perpendicular thereto, which we may call the velocity pair. Both these pairs rotate together at some uniform angular velocity ω, which will in general differ from that which would hold for unretarded motion ω_0, as in Fig. 14, or from that which would hold for retarded unreinforced motion, as in Fig. 15.

Considering the displacement pair, the first member is the elastic force OF, modified by the new elastic force OS, to OF', Fig. 16. The new virtual force of inertia is Of. Consequently

$$OF' + Of = 0 \quad \text{dynes}$$

or

$$-mar\epsilon^{j\omega t} + m\chi(r)\ \cos\theta\ \epsilon^{j\omega t} + m\omega^2 r\epsilon^{j\omega t} = 0 \qquad \text{dynes } \angle$$

whence

$$\omega = \sqrt{a - \cos\theta \cdot \chi(r)/r} \qquad \text{radians/sec.} \quad (17)$$

Considering the velocity pair, the first member is

$$Of' = -\Gamma v = -2m\gamma v = -j2m\gamma\omega r\epsilon^{j\omega t} \qquad \text{dynes } \angle$$

The second member is the T component:

$$OT = -jm\chi(r)\ \sin\theta \cdot \epsilon^{j\omega t} \qquad \text{dynes } \angle$$

For equilibrium

$$Of' + OT = 0$$

or

$$-j2m\gamma\omega r\epsilon^{j\omega t} - jm\chi(r)\ \sin\theta \cdot \epsilon^{j\omega t} = 0$$

From which

$$\chi(r)\ \sin\theta = -2r\gamma\omega \qquad \text{dynes} \qquad (18)$$

and

$$\omega = \sqrt{\omega_0{}^2 + \gamma^2 \cot^2\theta} + \gamma \cot\theta \qquad \text{radians/sec.} \quad (19)$$

It follows that ω, the new angular velocity under reinforcement, is independent of the force function $R = m\chi(r)$, and depends only on the natural angular velocity ω_0, the phase retardation θ of the restoring force and the magnitude of the damping coefficient γ. Some curves of ω as a function of θ for four particular values of Γ between 50 and 500 dynes per cm. per sec.; *i. e.*, of γ between 500 and 5,000 dynes per cm. per sec. and per gm., are given in Fig. 17. It may be seen that for all values of the damping, $\omega = \omega_0$ for $\theta = 270°$. That is, the angular velocity of reinforced motion is the same as that of unretarded motion when the restoring force is applied at 270° of phase lag, or exactly in phase with the velocity, as seen in Fig. 16. If the phase retardation θ is between 180° and 270°, the new angular velocity will be greater than the natural angular velocity ω_0; but if θ is between 270° and 360°, must be less than ω_0.

Applying the above principle to the corresponding case of reinforced vibration, by taking the projections or real parts of the rotating vectors, it follows that if any automatically reinforced vibrating system, such as an electromagnetic bell, electromagnetic tuning fork, or humming telephone, is propelled by an elastic force proportional to the displacement, reinforced by a cyclic force some function of the displacement, and damped by a force proportional to the velocity, it is subject to equations (17), (18) and (19) which appear to be new.

In the series of measurements on the humming telephone above

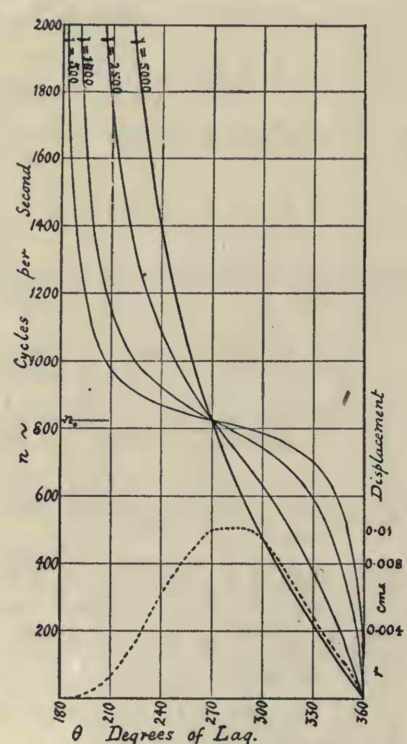

FIG. 17. Reinforced Frequency in Relation to the Phase of the Reinforcement.

outlined, the force function $R = m\chi(r)$ was not measured. The restoring electromagnetic force on the receiver diaphragm, due to the action of the transmitter, will manifestly diminish when the tube-length is increased. For a fixed tube-length, moreover, it cannot increase indefinitely in simple proportion to the displacement of the diaphragm, or to its amplitude of vibration. If we assume provisionally that R increases as the square root of the amplitude of receiver-diaphragm vibration; so that for a fixed tube-length, $R = bm\sqrt{r}$; *or* $\chi(r) = b\sqrt{r}$; where b is the numerical constant 1.036×10^6 dynes per gm. and per $\sqrt{\text{cm.}}$; then for the example already considered, if $\Gamma = 100$; or $\gamma = 1{,}000$, we find:

The displacement $r = 0.01$ cm. when $\theta = 270°$.

The reinforced angular velocity is $\omega = 5{,}184$ radians per second; $n = 825 \sim$.

The maximum cyclic values of the vibratory—

velocity $v =$	51.84 cm. per sec.
acceleration $c =$	268,740 cm. per sec.^{2}
damping force $Of' =$	5,184 dynes.
restoring force $OT =$	5,184 dynes.
elastic force $OF' =$	13,435 dynes.
inertia force $Of =$	13,435 dynes.

As the phase θ of reinforcement changes from 180° to 360°, the line of $\gamma = 1{,}000$ in Fig. 17 shows the change in frequency; while the dotted line indicates the computed amplitude of vibration, which reaches a maximum near 280°.

According to the theory, therefore, if the phase of the displacement is 270° behind the displacement of the receiver diaphragm, the reinforced frequency coincides with the natural frequency. This condition is substantially borne out in all of the observations. For example, in Fig. 2, taking the pitch line No. 1, with a natural frequency of $n_0 = 825 \sim$ and a sound-velocity in air of 33,000 cm. per sec., the wave-length $\lambda = 33{,}000/825 = 40$ cm. corresponding to 360° of phase. A lag of 270° would be represented by 30 cm.; so that we should expect the reinforced frequency to be $825 \sim$ at 30 cm. of tube-length, and at every 40 cm. beyond; *i. e.,* in accordance with the series $30 + 40m$, as was substantially observed. Moreover, by reversing the receiver terminals, the phase of the reinforcement is necessarily changed 180°; so that with this change of connection, 270° of phase lag would be altered to 90° of phase lag, or 10 cm. of tube-length. The natural frequency of $825 \sim$ should then occur in conformity with the series $10 + 40m$ cm., as was substantially observed.

When the phase retardation θ of the restoring force (Fig. 16) is less than 270°, we should expect, according to the theory, that the pitch should rise; because the elastic resilience of the diaphragm is virtually increased by the OS component of the new force, and when $\theta > 270°$, on the contrary, the pitch should fall. This was always the case in the observations. We have to bear in mind, however, that with any given tube-length, an alteration of pitch involves a change of wave-length, and therefore a change of phase in transmission through the air-column, besides any electrical change in phase due to change in current frequency. In Figs. 3, 4 and 6, the sloping dotted lines are drawn to indicate constant acoustic phase retardation of 270° for all of the frequencies within the range considered. Taking, for instance, Fig. 6, the break at P occurred 103° in phase from the dotted line of 270°, and the return at S occurred 77° in phase from the dotted line. According to the theory, assuming no electric change of phase, each of these angles should be something less than 90°, since the phase retardation must be something more than 180° on the side of increasing pitch. The discrepancy here is not serious; for the mean of the two angles is 90°. At T and W, however, the corresponding angles are 153° and 119°, with a mean of 136°, which should be something less than 90°, a greater divergence from the theory than observation errors can explain. While, therefore, the theory accounts for all of the experimental results in a general way, it can only be regarded as a first approximation. For example, it is possible that superposed harmonic currents might have to be considered; or that in estimating the damping forces, the inclusion of higher powers of the velocity than the first might be necessary.

Setting aside unexplained deviations, as the tube-length is shortened from a point of $\theta = 270°$, the phase retardation of the electromagnetic reinforcement on the receiver diaphragm is diminished. This causes the pitch to rise, and incidentally readjusts the phase change to a lower value than if the pitch were kept steady. The amplitude of vibration diminishes until the diaphragm suddenly selects a lower pitch for the same tube-length, to which the amplitude will be greater. In other words, the receiver diaphragm automatically seeks to maintain the greatest amplitude that the conditions of reinforcement will permit. If a lower tone, with a phase lag θ more than 270°, will give more amplitude than the higher note to which it has been driven, with θ less than 270°, it will break pitch downwards. This process will continue down to the first wave-length of tube, or 40 cm. in the case examined. For connection *I* of the receiver, it can break to no lower note after passing 270°, and the tone will rise to such a pitch that the amplitude becomes insufficient to excite the transmitter, so that silence should ensue at or near the length 12 cm., as actually observed in Fig. 3. The curve *I* of frequency between 12 and 50 cm. accords fairly well with the curve $\gamma = 1{,}000$ in Fig. 17.

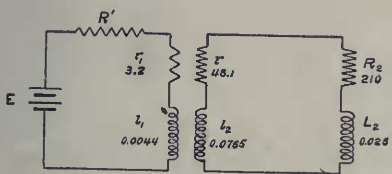

FIG. 18.

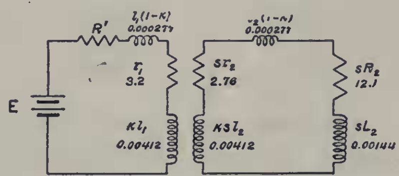

FIG. 19.

FIG. 18. Diagram of Electrical Connections with Step-up Induction Coil.
FIG. 19. Equivalent Diagram of Connections, with Level Induction Coil.

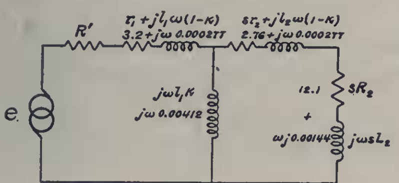

FIG. 20.

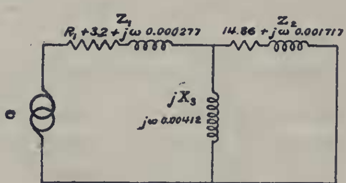

FIG. 21.

FIGS. 20 AND 21. Equivalent Conductive Connections with Level Induction Coil and Alternating E.M.F.

With reference to the influence of capacity in the secondary circuit, Figs. 18 to 21 show the successive steps by which the secondary circuit may be treated as a conductive branch of the primary circuit. Using the constants given in Table I., ignoring any capacity existing between the windings of the coils, and assuming that the effect of the transmitter in the primary circuit is equivalent to an alternating e.m.f. *e*, working through a transmitter resistance R_1 of 50 ohms, we find a coupling coefficient for the coil of $K = 0.937$, and an inductance ratio $S = 0.0575$ between primary and secondary windings. Proceeding in this way, the following table has been arrived at, giving the conductances which when multiplied by the equivalent transmitter e.m.f. yield the current strength in the secondary circuit of Fig. 21.

TABLE III.

Frequency Cycles per Second. n.	Angular Velocity Radians per Second. ω.	Capacity in Secondary Circuit. μf.	Conductance of Secondary. Mhos.
637	4,000	$\propto$	0.009 $\underline{/23^\circ}$
796	5,000	$\propto$	0.0087 $\underline{/17^\circ}$
956	6,000	$\propto$	0.0093 $\underline{/12^\circ}$
637	4,000	0.2	0.0054 $\underline{\backslash 140^\circ}$
796	5,000	0.2	0.0095 $\underline{\backslash 123^\circ}$
956	6,000	0.2	0.11 $\underline{\backslash 115^\circ}$

Although the assumptions employed do not anticipate a high degree of accuracy in the conclusions above tabulated, yet we may safely infer that when no condenser is used in the secondary circuit (μf. $= \propto$), the secondary current will lead the impressed primary e.m.f. by a small angle, and this current will have substantially the same strength and phase for all frequencies between 600 ~ and 1,000 ~. When, however, a condenser of 0.2 μf. is inserted in the secondary circuit, the current in the receiver will be advanced in phase about 110° or nearly a third of a cycle; while the strength of this current will be considerably greater at the higher frequencies than at the lower frequencies.

Since the total lag in phase of the restoring electromagnetic force behind the displacement of the receiver diaphragm includes (1) the electric current lag; (2) any hysteretic electromagnetic lag in the receiver cores; (3) any mechanical inertia lag of the transmitter diaphragm; (4) the acoustic lag in the air column of the tube; it follows that the total lag with a condenser of 0.2 μf. should be about 110° less than with short-circuited condenser; while the higher frequency notes should be favored, and the lower frequency notes disfavored. Fig. 9 shows that both these effects took place, the acoustic lag had to be increased by about 12 cm., or about 110°, in order to produce mean frequency, and compensate for the current lead.

Also the range of frequency is moved bodily towards higher notes.

All of the experimental series of observations appear to be accounted for and explained by the above theory to a first approximation; although in matters of quantitative detail there remains much room for further development.

In conclusion, the authors desire to express their indebtedness to the Western Electric Co. for the loan of apparatus used in the tests.